Published by Institute of Physics Publishing for SISSA/ISAS
Received: *October 3, 2002*
Revised: *November 27, 2002*
Accepted: *November 28, 2002*

Gravitational radiation of rolling tachyon

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Abstract: In this paper we compute the radiation of the massless closed string states due to the non-vanishing coupling to the rolling open string tachyon with co-dimensions larger than 2, and discuss the effect of back reaction to the motion of the rolling tachyon. We find that for small string coupling, the tachyonic matter remains in the late time, but it will completely evaporate away over a short time of few string scales if the string coupling is huge. Comment on the implication of our results to the g-theorem of boundary conformal field theory is given.

Keywords: Tachyon Condensation, Classical Theories of Gravity.

http://jhep.sissa.it/archive/papers/jhep112002050/jhep112002050.pdf

JHEP11(2002)050

Contents

1. Introduction

The tachyon problem remained a significant problem in the past, ever since the bosonic string was found to contain a tachyon in its spectrum. It appears to be a universal feature of string theory when supersymmetry is broken at the tree level, thus the study of the fate of tachyon is an important problem. In the past few years string theory has gained tremendous progress in understanding the symmetry nature of the theory itself, along the way, both the stable and unstable static D-branes were found. Especially, the unstable D-brane solutions provide an arena for studying the tachyon condensation dynamics, and many physical issues about the endpoint of the tachyon condensation have been clarified. In contrast, there is little understanding about the dynamical nature of the tachyon motion itself, namely, its time-dependent behavior and the couplings to the (time-dependent) background spacetime.

In analogue to the difference between the electrostatics and electrodynamics, a novel feature of these time-varying solutions is that they can produce the gravitational radiations which will then react back to the motion of the tachyon and the background geometry. Once the radiation rate of a rolling tachyon is computed, one can plug it back to the equation of motion for the tachyon to compute the back-reaction. We shall show in this paper that radiation is a generic feature of a rolling tachyon, and for simplicity we will consider a unstable D-brane embedded in spacetime with codimension equal to or greater than 3. Other cases require separate study. We shall show that once the string coupling is increased, the radiation is comparable to the initial energy stored in the unstable D-brane, and it is possible for the D-brane to complete release its energy into the bulk if the coupling is sufficiently large. In this case, the decay time is the string scale. However, we will see that the numerical factors require a large string coupling constant in order to have significant energy loss, and the complete decay of the unstable brane, if possible, may takes a longer time.

We shall not consider the interesting case when the transverse space is compactified. This problem is interesting in the cosmological context, when one attempts to take the rolling tachyon into account in any study of a cosmological scenario, or to take the tachyon as an inflaton, since the reheating problem was pointed out to be a severe problem in some tachyon-driven inflation scenario, and the decay of tachyon may be of significant importance in resolving this problem. Also, the case of compactified transverse space is related to the case of D-brane with less than 3 transverse space.

A related problem is the S-brane, recently studied by Gutperle and Strominger in [1]. They have proposed a new type of branes with spacelike worldvolumes which is related to the dynamical process of the open string tachyon condensation. The conjectured supergravity solutions of the above "S-branes" are constructed in [1] and [2] based on the isometry of the brane geometry. The S-brane configuration is essentially different from Sen's rolling tachyon which describes the motion of the unstable D-brane tachyon in open string (field) theory. The difference lies in the initial conditions. In case of S-brane, some fine tuned initial conditions are required. Nevertheless, one can see the radiation effects in the S-brane configuration. See [1] and [14] for the related discussions and the new developments.

Both S-brane and the rolling tachyon can be described by the on-shell string dynamics, and the corresponding boundary states, which encode both the open and closed string dynamics, are constructed in [1] and [3]. However, the interactions between the rolling tachyon and the closed string modes by using these boundary states has not been extensively done yet, see the last three references in [4] for related consideration.

The other interesting point in considering the gravitational radiation is that the energy density stored in the brane can be interpreted as the boundary entropy g [13], therefore the energy loss due to the radiation implies decrease of g as tachyon rolls, this manifest a generalized g-theorem in the context of boundary conformal field theory, since coupling to closed strings is described as loop effects in the open string picture.

The paper is organized as following: In the next section we first generalize Sen's rolling tachyon for the D25-brane to the one with transverse directions, and calculate the couplings between rolling tachyon boundary states and the massless closed string modes, then obtain the linear fluctuations representing the gravitational radiations in the asymptotic regions. In the section 3, we will calculate the radiation power accounting for the energy carried away to asymptotic infinity by the gravitational radiations. In section 4, we will use the energy loss rate calculated in section 3 as the back reaction to the time evolution of the rolling tachyon. In section 5, we will conclude with comments and discussions, especially on the implication of the gravitational radiation to a generalized g-theorem of the boundary conformal field theory when taking into account the loop effects in open string picture, which is the re-interpretation of the coupling to closed string, especially to the massless mode, i.e. gravitational radiation.

We perform all calculations in the bosonic string theory, ignoring the existence of the closed string tachyon. Calculations in super string theories are similar so we shall not do them here.

It remains to clarify the physical significance of the exponential growth of massive string states in the boundary state, a phenomenon observed in a couple of references in [4].

2. Gravitational radiations from rolling tachyon

In [3] Sen considered the rolling tachyon by Wick rotating the usual marginal operator $T(X)$ of the boundary conformal field theory(BCFT)

$$T(X) = \lambda \cos(X) \longrightarrow T(X^0) = \lambda \cosh(X^0)\,, \tag{2.1}$$

which describes the rolling of the tachyon initially displaced away from the top of the potential by $T(X^0 = 0) = \lambda$. After using some tricks in BCFT, the boundary state of the rolling tachyon up to the $(1,1)$-level in the time direction is constructed and given as [3]

$$|B\rangle = \mathcal{N}\left[f(X^0) + \alpha^0_{-1}\bar{\alpha}^0_{-1} g(X^0)\right] \delta^{d_t}(x^\alpha) \exp\left\{-\sum_{n=1}^{\infty} \alpha^{\mu\dagger}_n C_{\mu\nu} \bar{\alpha}^{\nu\dagger}_n\right\} |0\rangle \tag{2.2}$$

with $C_{\mu\nu} = (\delta_{\alpha\beta}, -\delta_{ij})$ and $|0\rangle \equiv |k=0\rangle$ the zero-momentum state. We have generalized the rolling tachyon on the D25-brane in [3] to the one on the Dp-brane. For this, the indices of the first entry of $C_{\mu\nu}$ run along the spatial transverse directions, and the ones in the second entry run along the longitudinal directions. Here we assume the spacetime dimension be d and the rolling tachyon be living on the spatial hypersurface of the Dp-brane so that the dimensions of the spatial transverse directions be $d_t = d - p - 1$.

Computation of the gravitational radiation of a rolling tachyon in a type-II string theory is similar to what we shall do here and we will not perform it in this paper.

The rolling profiles are determined in [3] as

$$f(x^0) = \frac{1}{1 + e^{x^0}\sin(\lambda\pi)} + \frac{1}{1 + e^{-x^0}\sin(\lambda\pi)} - 1\,, \qquad f(x^0) + g(x^0) = \cos(2\lambda\pi) + 1\,. \tag{2.3}$$

by treating boundary state $|B\rangle$ as the source to the closed string field $|\Phi_c\rangle$ satisfying the linearized equation of motion $(Q_B + \bar{Q}_B)|\Phi_c\rangle = |B\rangle$ where Q_B is the BRST operator. From the conservation equation $(Q_B + \bar{Q}_B)|B\rangle = 0$ one can read out the stress tensor which turns out to be

$$T_{00} = T_p(f(x^0) + g(x^0))\delta^{d_t}(x^\alpha)\,, \qquad T_{ij} = -2\delta_{ij} T_p f(x^0)\delta^{d_t}(x^\alpha)\,, \qquad T_{0i} = 0\,, \tag{2.4}$$

and

$$T_{\alpha\beta} = T_{\alpha i} = T_{0\alpha} = 0\,, \tag{2.5}$$

yielding the constant energy density($\sim$ brane tension T_p) and pressureless tachyonic matters in the late time [3].[1]

At this point, it is interesting to point out that the S$(p-1)$-brane boundary state up to the $(1,1)$-level in the time direction also takes the form of (2.2) but with the rolling profiles

$$f(x^0) = -g(x^0) = \delta(x^0)\,. \tag{2.6}$$

This implies zero energy density for the S$(p-1)$-brane.

[1] In [3] the third equation of (2.4) is absent since there is no transverse directions there, we will see later that this is the case if there are some.

Since the boundary state encodes all the closed and open string dynamics about the rolling tachyon, we can consider the coupling between the massless closed string modes and the boundary state (2.2) to extract the gravitational perturbations in the asymptotic flat region as in the case of the static D-brane given in [6]. Moreover, because the source is time-varying, one would expect it produce the gravitational radiation once the string coupling is turned on. This radiation will then back react to the tachyon motion and the background geometry.

Following [6], the asymptotic metric and dilaton perturbations can be read off from the current

$$J_{MN} = \langle V_c|D|B\rangle \tag{2.7}$$

where

$$|V_c\rangle = \alpha_{-1}^M \bar{\alpha}_{-1}^N |k_0, k_\alpha, k_i\rangle \tag{2.8}$$

is the massless closed string vertex operator, and

$$D = \frac{1}{2\pi}\int_{|z|\leq 1} \frac{d^2 z}{|z|^2} z^{L_0 - 1}\bar{z}^{\bar{L}_0 - 1} \tag{2.9}$$

is the closed string disk propagator. The indices M, N run from 0 to $d-1$.

The straightforward calculation tells us

$$J^{MN} = \begin{cases} \Delta(k)\tilde{g}(k_0)\,, & M = N = 0\,, \\ \Delta(k)C_{\mu\nu}\tilde{f}(k_0)\,, & M, N \neq 0 \end{cases} \tag{2.10}$$

where $\tilde{f}(k_0)$, $\tilde{g}(k_0)$ are the fourier transformation of the profiles $f(x_0)$, $g(x_0)$, and

$$\Delta(k) = \mathcal{N}\left(\frac{-1}{(k_0 + i\epsilon)^2 - k_\alpha^2}\right)\delta^p(k_i)\,. \tag{2.11}$$

is the spacetime propagator of the massless closed string modes in k-space. Take $\epsilon > 0$ for the retarded propagator. Note that the $\delta^p(k_i)$ factor in (2.11) implies no wave propagation along the longitudinal directions.

Using the convention in [6] for the polarization projection of (2.10), we obtain the asymptotic fluctuations of the dilaton and the metric:

$$\frac{1}{\kappa}\phi = -\Delta(k)\left[(1+2b)\tilde{g}(k_0) + (2p - d + 3 + 2b)\tilde{f}(k_0)\right], \tag{2.12}$$

$$\frac{1}{2\kappa}h_{00} = -\frac{\Delta(k)}{d-2}\left[(3 - d + 2b)\tilde{g}(k_0) + (2p - d + 3 + 2b)\tilde{f}(k_0)\right], \tag{2.13}$$

$$\frac{1}{2\kappa}h_{\alpha\beta} = \frac{\Delta(k)}{d-2}\left[(1+2b)\tilde{g}(k_0) + (2p + 1 + 2b)\tilde{f}(k_0)\right]\delta_{\alpha\beta}\,, \tag{2.14}$$

$$\frac{1}{2\kappa}h_{ij} = \frac{\Delta(k)}{d-2}\left[(1+2b)\tilde{g}(k_0) + (2p - 2d + 5 + 2b)\tilde{f}(k_0)\right]\delta_{ij}\,, \tag{2.15}$$

where b is an arbitrary gauge parameter due to the freedom in choosing the polarization vector, i.e. $b = k_0 l_0$ in [6]. These fluctuations are propagating along the transverse directions, they are the asymptotic gravitational radiations from the rolling tachyon.

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These fluctuations can also be understood as the linear perturbation around the flat spacetime of the following dilatonic gravity[2] in the Einstein frame [6]

$$S = -\frac{1}{2\kappa^2}\int d^dx\sqrt{-g}\left[R + \frac{1}{d-2}(\partial\phi)^2 + \cdots\right] \tag{2.16}$$

where we omit the couplings of the dilaton to the form fields.

We can read off the source stress tensor from the linearized Einstein equation

$$\bar{h}_{MN,K}^{\ \ \ \ ,K} - \bar{h}^K_{M,KN} - \bar{h}^K_{N,KM} + \eta_{MN}\bar{h}^{KL}_{,KL} = -2\kappa^2 T_{MN}\,, \tag{2.17}$$

where $\bar{h}_{MN} \equiv h_{MN} - \frac{1}{2}\eta_{MN}h$, and from (2.13) to (2.15) we have

$$\frac{1}{2\kappa}\bar{h}_{00} = \Delta(k)(1+b)\left[\tilde{g}(k_0) + \tilde{f}(k_0)\right], \tag{2.18}$$

$$\frac{1}{2\kappa}\bar{h}_{\alpha\beta} = -\Delta(k)b\left[\tilde{g}(k_0) + \tilde{f}(k_0)\right]\delta_{\alpha\beta}\,, \tag{2.19}$$

$$\frac{1}{2\kappa}\bar{h}_{ij} = -\Delta(k)\left[b(\tilde{g}(k_0) + \tilde{f}(k_0)) + 2\tilde{f}(k_0)\right]\delta_{ij}\,. \tag{2.20}$$

Surprisingly they are in the nice forms independent of the dimensional parameters p and d, moreover, all the b-dependence comes with the $f+g$ which is constant in time and therefore will not contribute to the radiations.

In order to read off the source stress tensor, it is better to have the Lorentz condition $k_M\bar{h}^M_N = 0$, however, it is ambiguous to see if the Lorentz condition holds because it will involve the quantity $k_0\Delta(k)(\tilde{f}(k_0) + \tilde{g}(k_0)) \sim k_0\frac{1}{(k_0+i\epsilon)^2 - k_\alpha^2}\delta(k_0)$ for the rolling tachyon, which can be either zero or blow up if the mass-shell condition is also imposed. But the pole in $\Delta(k)$ can be properly regularized by the usual Green's function method so that we can set it to zero and the Lorentz condition holds. The linearized Einstein equation is then simplified to

$$\bar{h}_{MN,K}^{\ \ \ \ ,K} = -2\kappa^2 T_{MN}\,. \tag{2.21}$$

It is then straightforward to get T_{MN} which turns out to be nothing but (2.4) and (2.5) if we set

$$\mathcal{N} = \kappa T_p\,. \tag{2.22}$$

Note that we have taken the $b=0$ gauge for simplicity.

The normalization constant $\mathcal{N}$ for the static brane is given in [6] from the factorization of the scattering amplitude. Moreover, $\mathcal{N} = \sqrt{\pi}/32 \sim 10^{-2}$ for $d = 26$ if we set the string scale $2\pi l_s$ to 1 and is independent of d_t. For only concern of the order of magnitude, we assume the normalization for the rolling tachyon has the same order. From now on we will set $2\pi l_s = 1$ such that $\kappa = \mathcal{N}/T_p = 2\pi g_s$ in which our $\mathcal{N}$ is different from the one in [9] by a factor of 1/2, this is because we have absorbed a factor of 1/2 into $\mathcal{N}$ in defining the boundary state, that is equivalent to halving the brane tension.

In summary: we have derived the source's stress tensor in the context of the linearized perturbation of the dilatonic gravity by starting from the coupling of the boundary state to

[2] We use the conventions of [8] for the curvature tensor, which is related to the others' by flipping the sign of the curvature tensor.

the massless closed string modes. Moreover, these asymptotic fluctuations are the classical gravitational radiations from the rolling tachyon. In the next section we will apply the above first order result and then consider the stress tensor carried by the asymptotic fluctuations in the second order of Einstein equation.

3. Instantaneous radiation power

Base upon the results in the previous section, in the following we will calculate the radiation power due to the asymptotic wave fluctuations. The radiation power per transverse solid angle measured at spatial infinity is given by

$$\frac{dP}{d\Omega_{d_t-2}} = \lim_{r\to\infty} r^{d_t-1}\hat{x}^{\alpha}\, t_{\alpha 0}\,, \tag{3.1}$$

where t_{MN} is the stress tensor for the gravitational or dilatonic waves carried by the asymptotic perturbations, and $r^2 = x_\alpha^2$, $\hat{x}^\alpha = x^\alpha/r$ where index α runs for only the transverse directions¡ since there is no wave propagating along the longitudinal directions, i.e. $k_i = 0$.

The stress tensor for gravity wave is encoded in Einstein equation of the second order in h_{MN}, that is,

$$t^{(g)}_{MN} = \frac{1}{2\kappa^2}\left[R^{(2)}_{MN} - \frac{1}{2}\eta_{MN}\eta^{KL}R^{(2)}_{KL}\right], \tag{3.2}$$

where the 2nd order Ricci tensor is given by [8]

$$\begin{aligned} R^{(2)}_{MN} = & -\frac{1}{2}h^{KL}[h_{KL,MN} - h_{ML,NK} - h_{kN,LM} + h_{MN,LK}] + \\ & +\frac{1}{4}\bar{h}^{K}_{L,K}\left[h^{L}_{M,N} + h^{L}_{N,M} - h^{,L}_{MN}\right] - \\ & -\frac{1}{4}\left[h^{M}_{K,L} + h^{,M}_{KL} - h^{M}_{L,K}\right]\left[h^{K,L}_{M} + h^{KL}_{,M} - h^{L,K}_{M}\right]. \end{aligned} \tag{3.3}$$

Note that the above second line vanishes if h_{MN} satisfies the Lorentz gauge condition. In the following we will stick to this gauge by setting $b = 0$. The restriction to the Lorentz gauge will not affect the physical result since the gauge parameter b is always associated with $f(x^0) + g(x^0)$ which is constant in time and will not contribute to the radiation power.

Even without knowing the explicit coordinate-space form of $\bar{h}_{MN}$ of (2.13)–(2.15) but just the fact that $\bar{h}_{00}$ is constant in time, after some lengthy calculation, we can obtain the following simple result

$$2\kappa^2 t_{0\alpha} = R^{(2)}_{0\alpha} = 2c_1\bar{h}^{(l)}_{,0\alpha}\bar{h}^{(l)} + c_1\bar{h}^{(l)}_{,0}\bar{h}^{(l)}_{,\alpha} + c_2\bar{h}^{(l)}_{,0}\bar{h}^{(0)}_{,\alpha}\,, \tag{3.4}$$

where

$$\bar{h}^{(l)} \equiv \frac{1}{p}\,\delta^{ij}\bar{h}_{ij}\,, \tag{3.5}$$

and the constants c_1 and c_2 are

$$c_1 = \frac{-1}{4}\frac{p(d-p-2)}{d-2} = \frac{-1}{4}\frac{(d_t-1)(d-d_t-1)}{d-2}\,, \tag{3.6}$$

$$c_2 = \frac{1}{4}\frac{p}{d-2} = \frac{1}{4}\frac{d-d_t-1}{d-2}\,. \tag{3.7}$$

Moreover, if $\bar{h}^{(l)}$ decays exponentially in time, which is the case for Sen's rolling tachyon profile, then we can drop the total derivative term with respect to time and get the simpler result

$$2\kappa^2 t_{0\alpha} = -c_1 \bar{h}^{(l)}_{,\alpha} \bar{h}^{(l)}_{,0} \,. \tag{3.8}$$

Up to now we are considering the gravitational radiations from the homogeneous time-varying source. As known in the 4-dimensional electrodynamics and gravity, relating to their tensor structures, there are only dipole electromagnetic radiation and quadrupole gravity wave. Naively we would not expect gravity wave for a homogeneous source, however, what we have considered is the the gravitational waves from the extended objects in the dilaton gravity in d-dimensional spacetime, therefore there is always the dilaton radiation which requires no multipole structure, also different dimensionality may require different multipole structure for the gravity wave. From our formula (3.8) we see that $c_1 = 0$ for $d = 4$ and $d_t = 3$ which agrees with the result in 4-dimensional spacetime. For the other cases there will be nonzero asymptotic gravity radiation power as we will see in the following.

In order to get the explicit form for $\bar{h}^{(l)}$ in the radiation power formula, one needs the coordinate representation of the retarded Green's function in the asymptotic region of the $d_t + 1$ spacetime. This is not seen in the standard textbook for general $d_t \neq 3$, so we derive it below.

Let's start with the k-space representation of the Green's function for $n+1$-dimensional spacetime

$$G(\vec{x}, t) = \int \frac{d^n k dk_0}{(2\pi)^{n+1}} \frac{e^{i\vec{k}\cdot\vec{x} - ik_0 t}}{(k_0 + i\epsilon)^2 - k^2} \,, \tag{3.9}$$

where $k = |\vec{k}|$.

After integrating out the k_0 we get

$$\int \frac{d^n k}{(2\pi)^n 2k} \, e^{i\vec{k}\cdot\vec{x} - ikt} \,, \tag{3.10}$$

and choose the spherical polar coordinates such that

$$d^n k = k^{n-1} \sin^{n-2}\theta d\theta d\Omega_{n-2} \,, \tag{3.11}$$

and

$$\vec{k}\cdot\vec{x} = kr\cos\theta \,. \tag{3.12}$$

Then the Green's function becomes

$$G(\vec{x}, t) = \frac{\Omega_{n-2}}{2(2\pi)^n} \int k^{n-2} I(kr, n) e^{-ikt} dk \,, \tag{3.13}$$

where

$$I(kr, n) = \int_0^\pi e^{ikr\cos\theta} \sin^{n-2}\theta d\theta = \sqrt{\pi}\, 2^{\frac{n-2}{2}} \Gamma\left(\frac{n-1}{2}\right) \frac{1}{(kr)^{\frac{n-2}{2}}} J_{\frac{n-2}{2}}(kr) \,, \tag{3.14}$$

In the above we have used the definition of the integral representation of the Bessel function $J_\nu(z)$.

Since we are only interested in the large r behavior of the Green's function, we can use the large $z = kr$ expansion of $J_\nu(z)$, namely,

$$J_\nu(z) = \sqrt{\frac{2}{\pi z}} \left\{ \cos\left(z - \frac{\nu\pi}{2} - \frac{\pi}{4}\right)\left[1+\mathcal{O}\left(\frac{1}{z^2}\right)\right] + \sin\left(z-\frac{\nu\pi}{2}-\frac{\pi}{4}\right)\mathcal{O}\left(\frac{1}{z}\right)\right\}. \tag{3.15}$$

Keep the leading term in the above, which turns out to be the only term contributing to the radiation power in the $r \to \infty$ limit, and plug it into (3.14) and (3.13) we get

$$G(\vec{x},t) \sim \frac{\Omega_{n-2}}{(2\pi)^n}\sqrt{\pi}\, 2^{\frac{n-3}{2}}\Gamma\left(\frac{n-1}{2}\right) r^{\frac{1-n}{2}} \int_0^\infty dk k^{\frac{n-3}{2}} e^{ik(r-t)}\,, \tag{3.16}$$

where we have dropped the overall constant phase and also the advanced part proportional to $e^{-ik(r+t)}$.

From the fact

$$4\pi \int_0^\infty dk e^{ik(r-t)} = \delta(r-t) \tag{3.17}$$

so that we have

$$4\pi \int_0^\infty dk k^m e^{ik(r-t)} = (i)^m \frac{d^m \delta(t-r)}{dt^m} \equiv (i)^m \delta^{[m]}(t-r)\,. \tag{3.18}$$

For $n \geq 3$ and odd,

$$G(\vec{x},t) = g_n r^{\frac{1-n}{2}} \delta^{[\frac{n-3}{2}]}(t-r)\,, \tag{3.19}$$

where $|g_n| = \frac{\Omega_{n-2}}{\sqrt{\pi}(2\pi)^n}\, 2^{\frac{n-7}{2}}\Gamma(\frac{n-1}{2}) = 2^{-(\frac{n+5}{2})}\pi^{-(\frac{n+2}{2})}$.

In comparison with the leading term in (3.19) which will yield finite radiation power, the higher terms of $1/(kr)$ in $J_\nu(kr)$ have more negative power r-dependence, therefore these terms do not contribute to the radiation power measured at spatial infinity.

For $n > 3$ but even, we need to evaluate

$$\int_0^\infty k^{m+\frac{1}{2}} e^{ik(r-t)} dk\,, \tag{3.20}$$

which has no simple form in the coordinate space. Nevertheless, it is a simple matter to compute the total radiation energy in k-space.

For $0 < n < 3$ the brane curves the transverse directions, so the asymptotic consideration will break down, one may follow the treatment in [10] for the cases with one or two transverse directions. For $n = 0$ there is no transverse directions, instead one can follow the consideration of the tachyon cosmology [5, 11] where the rolling tachyon matters act as the source to the evolution of the background geometry.

For concreteness, we consider the cases with $d_t \geq 3$ and odd in the following. Using the Green's function (3.19) we will get the leading term for the asymptotic fluctuations

$$\bar{h}^{(l)}(\vec{x},t) = 4\kappa^2 T_p g_{d_t} \frac{1}{r^{\frac{d_t-1}{2}}} f^{[\frac{d_t-3}{2}]}(t-r)\,. \tag{3.21}$$

Combining this with (3.8) and (3.1) we get the corresponding radiation power for gravity wave

$$P_g = 8c_1 \Omega_{d_t-2}\left[\kappa T_p \left|g_{d_t}\right| f^{[\frac{d_t-1}{2}]}(t-r)\right]^2. \tag{3.22}$$

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This is the energy loss rate measured at the spatial infinity, for the instantaneous one measured at the source we should take care of the retarded effect and replace the argument $t-r$ by $t-r+r=t$ in the above formula.

Similarly, we can calculate the instantaneous radiation power due to the asymptotic dilaton wave whose stress tensor is the standard one derived from action (2.16), that is

$$t^{(\phi)}_{MN} \equiv -2\frac{\delta L^{(\phi)}}{\delta g^{MN}} + g_{MN}L^{(\phi)} = \frac{1}{2\kappa^2}\frac{1}{d-2}[2\partial_M\phi\partial_N\phi - \eta_{MN}(\partial\phi)^2]\,. \tag{3.23}$$

From (2.12) and (3.19) the dilaton fluctuation in the coordinate space is given by

$$\phi(\vec{x},t) = \kappa^2 T_p(d-2d_t)\,\frac{1}{r^{\frac{d_t-1}{2}}} f^{[\frac{d_t-3}{2}]}(t-r)\,. \tag{3.24}$$

Using this we can get the instantaneous radiation power of dilatonic wave as

$$P_\phi = -\frac{(d-2d_t)^2}{d-2}\Omega_{d_t-2}\left[\kappa T_p\,|g_{d_t}|\,f^{[\frac{d_t-1}{2}]}(t)\right]^2 . \tag{3.25}$$

The total gravitational instantaneous radiation power is

$$P = P_g + P_\phi = -c_{d,d_t}\left[\kappa T_p\; f^{[\frac{d_t-1}{2}]}(t)\right]^2 , \tag{3.26}$$

where the overall constant

$$c_{d,d_t} = \frac{2(d_t-1)(d-d_t-1)+(d-2d_t)^2}{(d-2)(\frac{d_t-3}{2})!2^{d_t+4}(\sqrt{\pi})^{d_t+5}}\,. \tag{3.27}$$

Note that P is the radiation power per unit world-volume, we then check that its dimension is L^{-p-2} as expected. Moreover, for any physical parameters d and d_t, P is always negative, i.e. energy loss.

4. Back reaction on rolling tachyon

The energy loss due to the gravitational radiations will have the back reaction on the tachyon motion so that the energy of the rolling tachyon will be no longer constant in time.

We can see from (3.26) that the energy loss is independent of the string or gravitational coupling since $\kappa T_p = \mathcal{N}$. This justifies our far field approximation. However, the ratio of energy loss to the origin energy density stored in the brane is proportional to g_s, i.e. $\kappa = \mathcal{N}/T_p = 2\pi g_s$. This implies that the energy loss will be comparable to T_p if g_s is large enough, and be negligible if g_s is very small. We can estimate the order of g_s in order for this to be the case. By integrating P over all time we get ratio of the total energy density loss to T_p

$$\frac{\Delta\rho}{T_p} = -2\pi g_s\mathcal{N}\,\frac{c_{d,3}}{6}\,\left[a^6+9a^4-9a^2-1-12(a^4+a^2)\ln a\right]\,(a^2-1)^{-3} \tag{4.1}$$

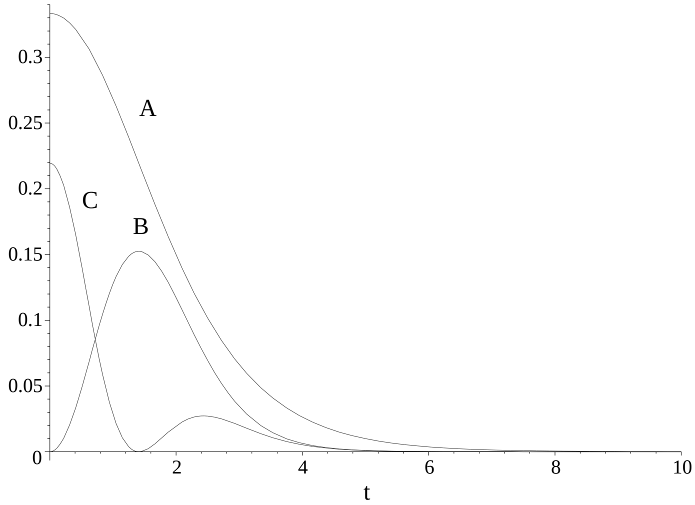

Figure 1: This plot shows that the function $\dot{f}(t)$ (the curve B) for $d_t = 3$ and $\ddot{f}(t)$ (the curve C) for $d_t = 5$ are localized in the early stage of the tachyon motion in contrast to the rolling profile $f(t)$ (the curve A). In the plot $\sin(\lambda\pi)$ is set to 0.5 and both $\dot{f}$ and $\ddot{f}$ have been scaled up by 10 times. This indicates that the gravitational radiation occurs in the early stage and lasts only for few stringy scale duration.

for $d_t = 3$; and

$$\frac{\Delta\rho}{T_p} = -2\pi g_s \mathcal{N}\,\frac{c_{d,5}}{30}\Big[a^{10} - 125a^8 - 350a^6 + 350a^4 + 125a^2 - 1 + \\ + 60(a^8 + 11a^6 + 11a^4 + a^2)\ln a\Big](a^2-1)^{-5} \tag{4.2}$$

for $d_t = 5$. The parameter a is the $\sin(\lambda\pi)$ in the definition of $f(t)$.

Both expressions for the energy loss are finite as long as $0 < a = \sin(\lambda\pi) < 1$, and the explicit value for $\mathcal{N}c_{d,d_t}$ shows that $g_s \sim 10^5$ for $d = 26$, $d_t = 3$ and $g_s \sim 10^7$ for $d = 26$, $d_t = 5$ in order for the energy loss to be comparable with the brane tension. Moreover, numerical plot of P shows that it has $\frac{d_t-1}{2}$ local maxima peaked around the order of string scale, and the distribution of P is more close to $t = 0$ than that for the rolling profile $f(t)$, see figure 1 for details. This implies that it is possible for the rolling tachyon to completely turn into the gravitational radiation if the string coupling is large enough, resulting in no remaining tachyonic matters in the late time. Of course, the energy loss should not be greater than the original energy stored in the brane to not violate the positive energy theorem in general relativity [12].

It is interesting to study the behavior of the above ratios in the $a \to 1$ limit.[3] Note that $a = 1$ or $\lambda = 1/2$ represents the closed string vacuum, and the initial energy density stored in the tachyon is $T_p(1 + \cos(2\lambda\pi)) = 2T_p(1 - a^2)$, which is vanishing in the $\lambda \to 1/2$ limit. In this limit we should expect a vanishing value for the above ratios (4.1) and (4.2) in accordance with the linear response theory. However, a naive inspection of (4.1) and (4.2) suggests that they diverge as $(a^2-1)^{-3}$ and $(a^2-1)^{-5}$ respectively. This is actually not

[3]We would like to thank the remarks of the JHEP's referee on this point.

true. After a careful treatment of the Taylor expansion of $\ln(a)$ in (4.1) and (4.2) up to the 5th and 7th order respectively, we find that both ratios vanish as $(a^2-1)^2$ and the corresponding ratio of the total energy loss to the original energy density vanishes actually as (a^2-1) which is consistent with the expectation of the linear response theory. This gives a nontrivial consistent check of our computation.

We should emphasize that the validity of our far field approximation should be independent of the magnitude of the gravitational coupling. This is in analogue to the case of black hole with a macroscopic huge mass, there we can trust our far field approximation for any mass. The only difference is that our far fields are not static. However, if g_s is large enough, then the radiation effect dominates the classical tachyon motion, and it is hard to describe the exact motion of the rolling tachyon even at late time. On the other hand, if g_s is small, then we can treat the gravitational radiation as a small external "frictional" force to the tachyon motion, then we can try to see the modified rolling behavior at the late time, namely, large time in comparison with the string scale.

In order to estimate the power loss rate in the late time, we can approximate the profile $f(t)$ by keeping the leading term in e^{-t} and get

$$f(t) \sim \cot(\pi\lambda)\cos(\pi\lambda)e^{-t}\,. \tag{4.3}$$

From this we know that the total radiation power in the late time behaves as

$$P \sim e^{-2t}\,, \tag{4.4}$$

which is negligible for large t. Therefore, for small g_s the overall physical picture on the back reaction of tachyon motion is that the energy of the tachyon decreases gradually in mild rate at the early stage and then comes to a constant again in the late time.

One can make the statement in more quantitative sense by invoking the effective field theory analysis of the rolling tachyon [3], which is described by the following DBI-type lagrangian [7]

$$-V(T)\sqrt{-\det(\eta+\partial T\partial T)} = -V(T)\sqrt{1-\dot{T}^2}\,, \tag{4.5}$$

where the tachyon potential $V(T) \sim e^{-T/2}$.

From (4.5) we can derive the energy density for the system, which is

$$\rho = \frac{V(T)}{\sqrt{1-\dot{T}^2}}\,. \tag{4.6}$$

If we neglect the gravitational radiations, ρ is conserved, then in the late time, the tachyon will have the solution in the following form

$$T = t + Ce^{-t} + \mathcal{O}(e^{-2t}) \tag{4.7}$$

so that $T \to \infty$ and $\dot{T} \to 1$. This field configuration result in the pressureless tachyonic matter in the late time.

By taking into account the small but non-vanishing gravitational radiation power, ρ is no longer conserved, instead

$$\frac{d\rho}{dt} = P\,. \tag{4.8}$$

Recall that $P \sim e^{-2t}$, it will not affect the leading term behavior of T in (4.7).

We can also consider the loss of the momentum density p_M due to the gravitational radiations by following the same procedure for the energy loss,[4] namely,

$$\frac{dp_M}{dt\; d\Omega_{d_t-2}} = \lim_{r\to\infty} r^{d_t-1}\hat{x}^\alpha\; t_{\alpha M}\,. \tag{4.9}$$

It is easy to see that there is no longitudinal momentum loss and the transverse momentum loss (recoil effect) has the same suppression factor e^{-2t} as for the energy loss in the late time with small string coupling. Therefore, the rolling tachyon will still form the pressureless matters in the late time, although the energy and transverse momentum stored in the rolling tachyon(or brane) is not conserved.

For sufficiently large string coupling constant or in the radiation active early stage, one needs to solve the equation of motion in a fully consistent way by taking into account the back reaction to the geometry, in order to have an exact profile. This is beyond the reach of this work. Moreover, since g_s is huge the quantum gravity effect could be quite relevant and the classical radiation would be less dominant. However, it is interesting to notice that the recoil effect is also huge, reflecting the necessity of studying the back reaction to the bulk geometry for highly strong coupling case even at the classical level.

5. Conclusion

In this paper, we have calculated the gravitational radiation power of the rolling tachyon, and consider its back reaction to the tachyon motion. We see that for Sen's smooth rolling profile, the perturbation calculation can be put into a nice framework of boundary state even though the background is time-dependent.

The total radiation energy, compared to energy stored initially in the unstable brane, is of order g_s. Thus naively one expects that when g_s is of order 1, the unstable brane completely decay into massless string modes, and the time it takes is the string scale. Surprisingly, this is not the case, since all numerical factors compromise to require a rather large coupling constant. When the back reaction is taken into account, the radiation can be larger, since the tachyon profile may change faster. It requires to solve the back reaction equation in order to see what really happens.

It is an interesting problem to calculate radiation when the codimension of the unstable brane is less than 3. In case of no codimension, we are dealing with a cosmological setting, all possible "energy release" takes the form of deforming the background metric in an uniform way. Besides, due to the form of the tachyon potential, it is difficult to get reheating in the rolling-tachyon driven inflation scenario [11]. If there is no transverse direction, the reheating problem remains. Otherwise, we would expect the energy release to the bulk if the compactified transverse space is sufficiently large, and the back reaction will modify the inflaton potential to solve the reheating problem. However, it requires more detailed studies to have a definite answer.

An implicit assumption for our calculation is that the rolling profile is non-singular. This is in contrast to the "rolling" profile of the Sp-brane where $f(t)=\delta(t)$, and it yields a singular instantaneous radiation power being proportional to $(\delta^{[\frac{d_t-1}{2}]}(t))^2$, implying that

[4]We thank C.-S. Chu for the discussion on this point.

the perturbation framework breaks down. This is not surprising since the Sp-brane is "created" and "annihilated" in a single moment, the energy profile is infinite which should excite all the stringy modes and put the string theory in the Hagedorn phase where both the quantum and thermal effects are essential. Some report in progress along line can be found in the recent paper by Strominger [14].

The final point we would like to mention is that our result implies a connection of the closed string radiations to the g-theorem of the BCFT. As pointed out in [13] the tension of the unstable brane can be interpreted as the boundary entropy g which counts the dimensions of the Hilbert space of BCFT. If one turns off the string coupling, there is no gravitational radiation and the energy density stored in the brane remains constant, yields no RG flow as the tachyon rolls. Once the string coupling is turned on, we see that the energy density of the brane is decreasing as tachyon rolls, this is a signature that the boundary entropy g is decreasing even we are only turning on the marginal boundary deformation. However this effect comes from interaction with closed strings, manifests itself as loop effects in the open string picture, thus does not contradicts Sen's result. The loop effects can be summarized at the open string tree level (the disk BCFT), and implies that the bulk radiation plays the role of the relevant deformation for BCFT, and it characterizes the g-theorem. Moreover, the positive energy theorem of general relativity, especially the one for the Bondi mass [12] guarantee that g is always positive as one should expect from its definition. However, it remains a task to give a more precise map between the Bondi mass and the quantity in BCFT. On the other hand, the massive closed string modes will not result in the IR divergence in the loop effect of the open string, in accordance with the fact that they cannot carry energy flux to infinity, therefore, they are not the relevant deformation of BCFT.

Acknowledgments

We appreciate the participation of Chiang-Mei Chen and John Wang at the early stage of this project, especially for C.M.'s many useful discussions and comments. B.C. and F.L.L. would like to thank ITP, Beijing for her hospitality during their visit this summer. The work of B.C. was supported by Korea Science and Engineering Foundation (KOSEF) Grant R02-2002-000-00146-0.

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